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A pursuit differential game with Gronwall types of constraints on players' acceleration controls

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Assume that in the space R^n a controlled object P, called a pursuer, chases another object E, called an evader. Denote by x a state of the pursuer and by y that of the evader in R^n. Let the motion dynamics of the players be generated by the following differential equations, initial conditions and Gronwall type constraints (briefly, Gr-constraint):

$$P: \quad \ddot{x} = u, \quad x(0) = x_0, \quad \dot{x}(0) = x_1, \tag{1}$$

$$E: \quad \ddot{y} = u, \quad y(0) = y_0, \quad \dot{y}(0) = y_1, \tag{2}$$

where $x, y, x_0, y_0, x_1, y_1, u, v \in R^n,\ n \geq 2$, x_0 and y_0 are initial states of the players, and x_1 and y_1 are their initial velocity vectors, respectively. We suppose that $x_0 \neq y_0$ and $x_1 = y_1$.

u is the velocity vector of the pursuer and here the temporal variation of u must be a measurable function $u(\cdot) : [0, \infty) \to R^n$.We denote by U_{Gr} the set of all measurable functions $u(\cdot)$ satisfying Gr-constraint

$$|u(t)| \leq \rho_0 + \rho_1 t + k \int_0^t |u(s)|\, ds \tag{3}$$

Similarly, v is the velocity vector of the evader and here the temporal variation of v must be a measurable function $v(\cdot) : [0,\ +\infty) \to R^n$. We denote by V_{Gr} the set of all measurable functions $v(\cdot)$ satisfying Gr-constraint

$$|v(t)| \leq \sigma_0 + \sigma_1 t + k \int_0^t |v(s)|\, ds \tag{4}$$

where $\rho_0,\ \rho_1,\ \sigma_0,\ \sigma_1$ are nonnegative numbers and k is positive number.

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By virtue of the equations (1)-(2) each pair of $(x_0,\ x_1,\ u(\cdot))$ and $(y_0,\ y_1,\ v(\cdot))$ generates the trajectories of players' motions

$$x(t) = x_0 + x_1 t + \int_0^t (t-s)u(s))ds,$$

$$y(t) = y_0 + y_1 t + \int_0^t (t-s)v(s)ds.$$

The goal of the pursuer P is to capture the evader E i.e., achievement of the equality $x(t) = y(t)$ and the evader E strives to avoid an encounter.

Lemma 1. *(Gronwall's inequality). If*

$$|\omega(t)| \le \alpha + \int_0^t (\beta + \gamma\,|\omega(s)|)\ ds$$

then $|\omega(t)| \le \frac{\beta}{\gamma}(e^{\gamma t} - 1) + \alpha e^{\gamma t}$, *where* $\omega(t)$, $t \ge 0$, *is a measurable function, and* α, β *are given non-negative numbers and* γ *is a given positive number.*

Definition 1. In the pursuit problem (1)-(4), the function

$$u_{Gr}(t,v) = v - \lambda_{Gr}(t,v)\xi_0 \tag{5}$$

is called a *Gr-strategy* of the pursuer, where

$$\lambda_{Gr}(t,v) =< v, \xi_0 > + \sqrt{< v, \xi_0 >^2 + \mu(t)^2 - |v|^2},$$

where $|\mu(t)| \le \frac{\rho_1}{k}(e^{kt} - 1) + \rho_0 e^{kt}$, $\xi_0 = \frac{z_0}{|z_0|}$ and $< v, \xi_0 >$ is a scalar product of the vectors v and ξ_0 in the space R^n.

Proposition 1. *If* $\rho_0 \ge \sigma_0$, $\rho_1 \ge \sigma_1$, *then the function* $\lambda_{Gr}(t,v)$ *is continuous and non-negative for all* $(t,v) \in [0,\infty) \times \mathbb{R}^n$.

Definition 2. The Gr-strategy is called *winning* for the pursuer on the interval $[0, T_{Gr}]$ in the pursuit problem (1)-(4) if for every $v(\cdot) \in V_{Gr}$, there exists some moment $t^* \in [0, T_{Gr}]$ for which $x(t^*) = y(t^*)$ is obtained. Here the time T_{Gr} is called a *guaranteed capture time.*

Theorem 1. *If one of the following conditions holds: 1)* $\rho_0 > \sigma_0, \;\; \rho_1 = \sigma_1$*, 2)* $\rho_0 = \sigma_0, \;\; \rho_1 > \sigma_1$*, 3)* $\rho_0 > \sigma_0, \;\; \rho_1 > \sigma_1$*, then Gr-strategy* (5) *for the player* P *is winning on the interval* $[0, T_{Gr}]$ *in the game* (1)-(4)*, where* T_{Gr} *is the smallest positive root of the equation:*

$$|z_0| - \frac{A}{k^2}e^{kt} + \frac{B}{2}t^2 + \frac{A}{k}t + \frac{A}{k^2} = 0,$$

where $A = \frac{\rho_1 - \sigma_1 + k(\rho_0 - \sigma_0)}{k}, \;\; B = \frac{\rho_1 - \sigma_1}{k}$.

1. References

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