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On a pursuit problem with impulse control of the players in the presence of delay

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1. Statement of the problem

In the space $\mathbb{R}^m$, we consider a linear differential game described by a system of linear delay differential equations [1, 2]

$$\dot{z}(t) = Az(t) + Bz(t-h) + Cu(t) - Dv(t), \quad t \geq 0, \tag{1}$$

where $z(t) \in \mathbb{R}^m$, $m \geq 1$; A, B, C, D are constant square (or rectangular for C, D) matrices of orders $(m \times m)$, $(m \times m)$, $(m \times p)$, $(m \times q)$, respectively. The symbol h is a fixed positive number (the delay); $u\,(t)$ is a control of the pursuer, $v\,(t)$ is a control of the evader.

The pursuer's aim is, by appropriately chosen control u, to steer the trajectory of system (1) to a cylindrical terminal set M within finite time. The terminal set has the form

$$M = M_0 + M_1. \tag{2}$$

Here M_0 is a linear subspace of $\mathbb{R}^m$, and M_1 is a nonempty compact subset of a subspace L, where L is the orthogonal complement of M_0 in $\mathbb{R}^m$. Denote by π the operator of orthogonal projection from $\mathbb{R}^m$ onto L, and by e^{tA} the fundamental matrix of the homogeneous part of (1). Clearly $z \in M$ iff $\pi z \in M_1$.

Let $\{\tau_i\}_{i=1}^{\infty}$ be an increasing sequence of time moments without finite accumulation points (every compact interval $[a, b]$ contains finitely many τ_i).

Pursuer's admissible controls are impulse functions represented via Dirac delta [3]:

$$u(t) = \sum_{i=1}^{\infty} u_i \delta(t - \tau_i), \quad t \geq 0, \tag{3}$$

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where jump vectors $u_i \in P$, P is a nonempty compact subset of $\mathbb{R}^p$. The class of admissible evader controls consists of measurable functions $v(\cdot)$ such that

$$\int_0^\infty \|v(t)\|^2 dt \le \sigma^2 \tag{4}$$

for fixed $\sigma \ge 0$.

From these definitions the following problem arise.

Problem. For a given initial point $\varphi(0) \notin M$ determine conditions under which, for any admissible evader control $v(\cdot)$ (from the evader class), using the available information, one can construct instantaneous (impulse) pursuer controls $u(\cdot)$ at times τ_i (from the pursuer's class of impulses) such that the resulting trajectory $z(t)$ of system (1), starting from $\varphi(\cdot) \in X$, reaches the terminal set M in finite time.

The initial state for system (1) is an m-dimensional absolutely continuous function $\varphi(t) \in X$, defined on the interval $[-h, 0]$, where

$$X = \varphi(t)\colon z(t) = \varphi(t), \quad t \in [-h, 0], \quad \varphi(0) \in \mathbb{R}^m \setminus M. \tag{5}$$

By $K(t)$, $-\infty < t \le \tau$, —we denote the unique matrix function possessing the following properties [1,2]: a) $K(t) = \tilde{0}$ for $t < 0$, where $\tilde{0}$ is the zero matrix of order m; b) $K(0) = E$, where E —is the identity matrix of order m; c) the entries of the matrix $K(t)$, belong to the class $C^1[0, \tau]$; d) for $t > 0$, the matrix function $K(t)$ satisfies the matrix differential equation $\dot{K}(t) = AK(t) + BK(t-h)$.

2. Solution to problem

In this section, we consider the game (1) under the assumption that the pursuer employs impulsive controls represented by the Dirac delta functions $\delta(t - \tau_i)$ in the form (3), while the evader is allowed to apply a measurable control $v(t)$, $t \ge 0$, satisfying condition (4). These are referred to as the admissible controls of the pursuer and the admissible controls of the evader, respectively.

Let $\tau-$ be an arbitrary positive number and let $t \in [0, \tau]$. Introduce the following sets:

$$\widehat{W}_i(n, v(\cdot)) = \pi K(\tau_n - \tau_i) CP - \int_{\tau_{i-1}}^{\tau_i} \pi K(\tau_n - t) D v(t) dt,$$

$$W_i(n) = \bigcap_{v(\cdot)\in Q[\tau_{i-1},\tau_i]} \widehat{W_i}(n, v(\cdot)) = \pi K(\tau_n - \tau_i)CP \overset{*}{-} \Phi_i(n, \tau_{i-1}, \tau_i),$$

where the set $\Phi_i(n, \tau_{i-1}, \tau_i)$ is defined as

$$\Phi_i(n, \tau_{i-1}, \tau_i) = \Big\{ y \in L : y = \int\limits_{\tau_{i-1}}^{\tau_i} \pi K(\tau_n - t) D v(t) dt, \;\; v(\cdot) \in Q[\tau_{i-1}, \tau_i] \Big\},$$

$i \in N$, $i = 1, 2, \cdots, n$. Here $Q[\tau_{i-1}, \tau_i]$ denotes the set of all functions $v(\cdot)$ measurable on $[\tau_{i-1}, \tau_i]$ and satisfying the constraint

$$\int\limits_{\tau_{i-1}}^{\tau_i} \|v(t)\|^2 dt \le \sigma^2.$$

Assumption 1. *The sets* $W_i(n)$ *are nonempty for all* $n \in \mathbb{N}$ *and all* $i = 1, 2, \ldots, n$.

By virtue of Assumption 1, from each set $W_i(n)$ one can select an element $w_i(n)$. Denote by $\omega = \omega(n) = \{w_i(n)\}_{i=1}^n$ a collection consisting of the elements $w_i(n)$, $i = 1, 2, \cdots, n$. Fix this collection and, following the basic scheme of resolving functions, define

$$\xi[n, \varphi(\cdot), \omega] = \pi K(\tau_n - \tau_0)\varphi(0) + \int\limits_{-h}^{0} \pi K(\tau_n - t - h) B \varphi(t) dt + \sum_{i=1}^{n} w_i(n),$$

and introduce the scalar function $\widetilde{\lambda}_i(n, \varphi(\cdot), v(\cdot), \omega)$ as follows:

$$\widetilde{\lambda}_i(n, \varphi(\cdot), v(\cdot), \omega) =$$

$$= \sup\Big\{ \lambda \ge 0 : \lambda\Big[M_1 - \xi[n, \varphi(\cdot), \omega]\Big] \bigcap \Big[\widehat{W_i}(n, v(\cdot)) - w_i(n)\Big] \ne \emptyset \Big\}.$$

Let k denote the following quantity:

$$k = k(n, \varphi(\cdot), v(\cdot), \omega) =$$

$$= \min\left\{ j \in \{1, 2, \ldots, n\} : \sum_{i=1}^{j} \widetilde{\lambda}_i(n, \varphi(\cdot), v(\cdot), \omega) \ge 1 \right\}.$$

If the inequality inside the braces is not satisfied for any $j \in \{1, 2, \ldots, n\}$, then we set $k = n + 1$.

Now we define the resolving function $\lambda_i(n, \varphi(\cdot), v(\cdot), \omega)$ as follows [2]:

$$\lambda_i(n, \varphi(\cdot), v(\cdot), \omega) = \begin{cases} \widetilde{\lambda}_i(n, \varphi(\cdot), v(\cdot), \omega), & i = 1, 2, \ldots, k-1, \\ 1 - \sum\limits_{j=1}^{k-1} \widetilde{\lambda}_j(n, \varphi(\cdot), v(\cdot), \omega), & i = k, \\ 0, & i = k+1, \ldots, n. \end{cases}$$

Theorem 1. *Assume that for system* (1) *under the pursuer's impulsive control* (3)*, Assumption 1 is satisfied, the sets* M_1 *and* P *are convex, and* $N(\varphi(\cdot), \omega) < +\infty$ *for the initial state* $\varphi(\cdot) \in X$ *and some collection* ω*. Then the trajectory* $z(t)$ *of system* (1) *can be driven to the terminal set* M *at the moment* $t = \tau_{N(\varphi(\cdot),\omega)}$*.*

References

[1] *Bellman R., Cooke K.L.* Differential–Difference Equations. Academic Press, New York, 1963; Mir, Moscow, 1967.

[2] *Mamadaliev N.A., Mustapokulov Kh.Ya., Abdualimova G.M.* The method of resolving functions for solving a pursuit problem with integral constraints on player controls. Vestn. Udmurtsk. Univ. Mat. Mekh. Komp. Nauki. 2023. Vol. 33, No. 1, pp. 103–118. DOI: 10.35634/vm230107

[3] *Chikrii A.A., Matichin I.I.* Linear differential games with impulse control of players. Trudy Inst. Mat. Mekh. UrO RAN. 2005. Vol. 11, No. 1, pp. 212–224; Proc. Steklov Inst. Math. 2005. Suppl. 1, pp. S68–S81. https://www.mathnet.ru/rus/timm182